Green roofs to Mitigate the Urban Heat Island

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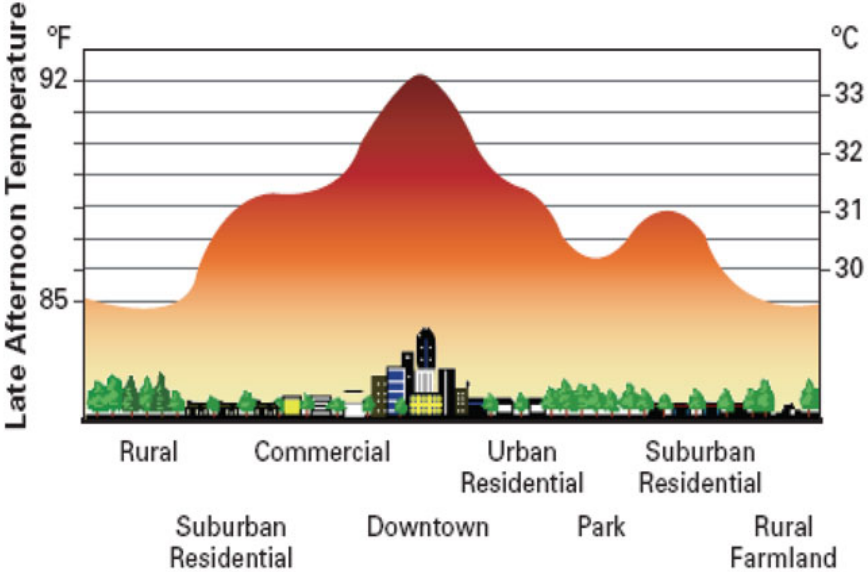
Introduction

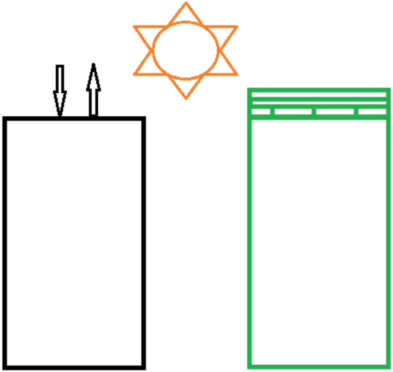
Urban Heat Island



Introduction

http://www.construyeargentina.com/wp-content/uploads/2013/06/green_roof3b.jpg

Mathematical Model



- The use of green roofs to reduce Urban heat Island
- **AIM:** How much energy is absorbed over one day?

Model

- Consider the heat equation

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

subject to the following BC's

$$T \longrightarrow T_{av}, \quad \text{as} \quad x \longrightarrow \infty$$

and

$$-k \frac{\partial T}{\partial x} = (1-\gamma) Q_{sun} + H(T - T_a) + \epsilon\sigma(T^4 - T_a^4) - \rho_w L_e \dot{m}$$

Heat flux = heat from sun + convective heat transfer from surface to air + radiative heat transfer to air + evaporative energy

Model

Nobody likes T^4 so we linearise

$$
\begin{aligned}
-k\frac{\partial T}{\partial x} &= (1-\gamma)Q_{sun} + (H + 4\epsilon\sigma T_a^3)(T - T_a) - \rho_w L_e \dot{m} \\
&= \left[(1-\gamma)Q_{sun} - (H + 4\epsilon\sigma T_a^3)T_a - \rho_w L_e \dot{m}\right] \\
&\qquad + \left[(H + 4\epsilon\sigma T_a^3)\right] T
\end{aligned}
$$

Model

Non-dimensionalise

$$\hat{T} = \frac{T - T_{av}}{\Delta T} \quad \hat{x} = \frac{x}{L} \quad \hat{t} = \frac{t}{\tau}$$

We choose to work over a time-scale $\tau = 3600$s

$$\frac{\rho c}{\tau}\frac{\partial T}{\partial t} = \frac{k}{L^2}\frac{\partial^2 T}{\partial x^2}$$

Hence $L = \sqrt{k\tau/\rho c} \approx 4$cm

Model

$$
\begin{aligned}
-\frac{k\Delta T}{L}\frac{\partial T}{\partial x} &= \left[(1-\gamma)Q_{sun} - (H + 4\epsilon\sigma T_a^3)(T_{av} - T_a) - \rho_w L_e \dot{m}\right] \\
&\quad + \left[(H + 4\epsilon\sigma T_a^3)\right]\Delta T . T
\end{aligned}
$$

Here we may choose ΔT as the whole of the first square bracket or $(1-\gamma)Q_{sun}$. Find $\Delta T \approx 10$K.
With whole of first bracket

$$
-\frac{\partial T}{\partial x} = 1 + \beta T
$$

where

$$
\beta = \left[(H + 4\epsilon\sigma T_a^3)\right]\frac{L}{k}
$$

Approximate Solution

A fundamental solution using Green's functions is given by:

$$T(x,t) = \frac{1}{\rho_c\sqrt{\pi k}}\int_0^t \frac{q(t')e^{-\dfrac{x^2}{4k'(t-t')}}}{\sqrt{t-t'}}dt'$$

where q represents the heat flux.
Then on the surface we have:

$$T(0,t) = T_s(t) = \frac{1}{\rho_c\sqrt{\pi k}}\int_0^t \frac{q(t')}{\sqrt{t-t'}}dt'$$

Approximate solution

We get :

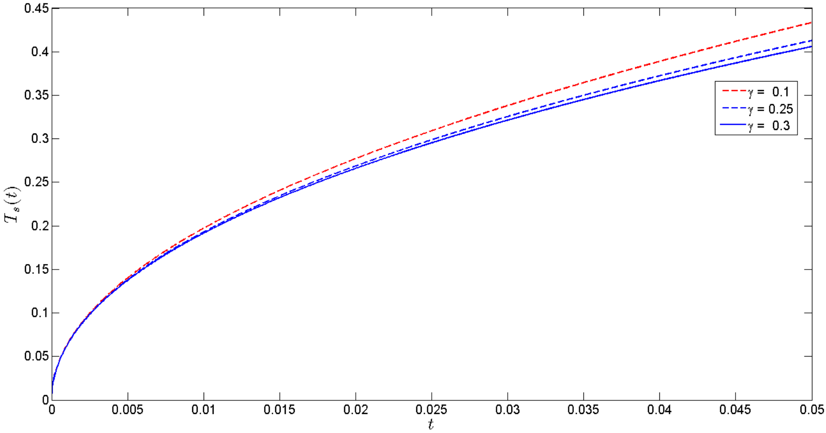
$$T_s(t) = \int_0^t \frac{1 + \beta T_s(t')}{\sqrt{t - t'}} dt',$$

An approximate solution is found by setting $\beta = 0$.

- $T_s = 2\sqrt{t}$, putting this expression back in the equation above and integrating for τ_s gives

$$T_s(t) = 2\sqrt{t} - 2\sqrt{2}\beta\sqrt{1 - t},$$

Approximate solution



Laplace Transform

An exact solution may be found to the system using Laplace transforms

$$T = \frac{1}{\beta}\left[e^{\beta(\beta(t-x))}\mathrm{erfc}\left(\frac{x}{2\sqrt{t}} - \beta\sqrt{t}\right) - \mathrm{erfc}\left(\frac{x}{2\sqrt{t}}\right)\right]$$

From this we see the exact behaviour of the temperature with space and time

The solution apears to be controlled only by β, but the temperature scale depends on values we wish to change

$$\begin{aligned}\Delta T &= \left[(1-\gamma)Q_{sun} - (H + 4\epsilon\sigma T_a^3)(T_{av} - T_a) - \rho_w L_e \dot{m}\right] L/k \\ \beta &= \left[(H + 4\epsilon\sigma T_a^3)\right] L/k\end{aligned}$$

$\beta \approx 0.03$

Laplace Transform

Can compare with surface solution by first setting $\beta \to 0$

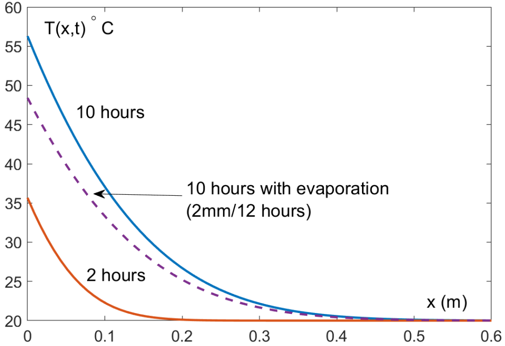
$$T = 2e^{-x^2/(4t)}\sqrt{\frac{t}{\pi}} - x\,\mathrm{erfc}\left(\frac{x}{2\sqrt{t}}\right) + O(\beta)$$

At $x = 0$

$$T = 2\sqrt{\frac{t}{\pi}} + \beta t + O(\beta^2)$$

Leading order $\propto \sqrt{t}$ as with previous. Easy to carry on the series

Solution



Temperature variation with air at 25C, average temperature 20C after 2 and 10 hours. Also shown is 10 hour curve with evaporation rate of 2mm/12 hours.

So, how can we use this?

The energy absorbed/unit area is

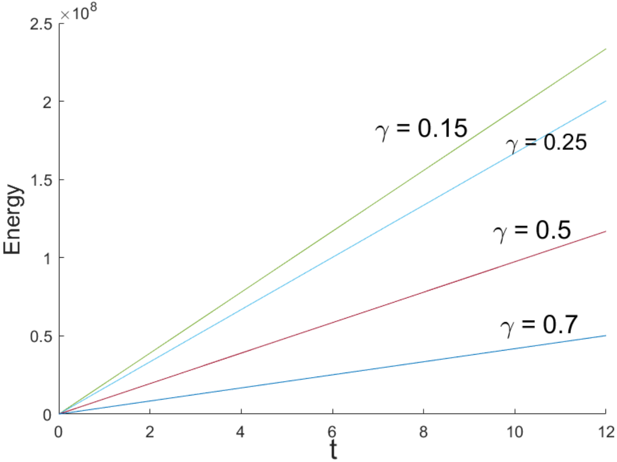
$$E = \rho c \int_0^\infty (T - T_{av})\, dx$$

We may plot this over time, to see the increase during the day or ...

Calculate E for different scenarios, changing albedo, adding evaporation etc

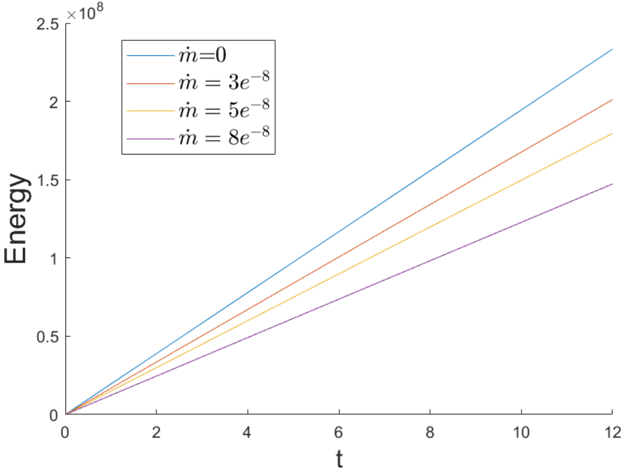
This will then tell us how much we may change the energy storage in a city under different scenarios.

Solution



Effect of changing albedo

Solution



Fixed albedo $\gamma = 0.15$, with evaporation

Conclusion

- Can find exact solution for temperature in a semi-infinite concrete slab (could also do finite).
- Green roofs/cool roofs do not have a noticeable effect in the street (if roof above 10m).
- Actual temperature profile in a city way beyond our skills but ...
- Exact solution allows us to find differences in absorbed energy and shows effect of albedo and evaporation.
  Most important terms in
  $\left((1-\gamma)Q_{sun} - (H + 4\epsilon\sigma T_a^3)(T_{av} - T_a) - \rho_w L_e \dot{m}\right) \sqrt{\tau/\rho ck}$
- Green roofs can reduce heat (soil layer will absorb much less heat). Evaporation also helps.
- High reflective/cool roofs may reflect more heat away.
- However, green roofs as well as reducing heat absorption also reduce CO2 and provide habitat for birds and insects.

The End